Abstract—This work focuses on the problem of jointly separating and reconstructing source signals from compressively sensed mixtures. The proposed approach combines the concept of blind compressive sensing (BCS) with the model of blind source separation (BSS). Specifically, the unknown source signals are assumed to admit some sparse representations with respect to an unknown but common dictionary. We propose an appropriate cost function for the problem together with an alternating minimization scheme for finding a solution. A fixed-point analysis as well as experiments on synthetic and on real data will be provided to demonstrate the efficiency and the convergence behavior of the proposed algorithm.

I. INTRODUCTION

Sparsity has demonstrated its performance in both signal separation and signal reconstruction. Well known techniques, such as morphological component analysis (MCA), cf. [1] and compressed sensing (CS), cf. [2], have been developed for the respective task in the case where some sparsity dictionary is given a priori. Recent work in [3] proposes a blended approach (compressively sensed BSS) to simultaneously separate and reconstruct source signals from compressively sensed linear mixtures. For all aforementioned methods, the prior knowledge of the sparsity dictionary is required. Recent works in [4] and [5] have developed two successful techniques, blind morphological component analysis (BMCA) and blind compressive sensing (BCS) respectively, where the sparsity dictionary is unknown. In this work, we extend the result from [3] via combining the concept of BCS with the model of BSS. The resulting algorithm is conceptually simple and alternates between three successive minimization steps, two of which allow for an explicit solution. We present a fixed-point analysis and numerical experiments that show the efficiency of our approach.

II. PROBLEM STATEMENT

Given the instantaneous linear BSS model as

\[ Y = SA, \]

where \( S = [s_1, \ldots, s_p] \in \mathbb{R}^{n \times p} \) denotes the data matrix of \( p \) sources with \( n \) samples (\( p \ll n \)), \( A = [a_1, \ldots, a_q] \in \mathbb{R}^{p \times q} \) is the mixing matrix of full rank, and \( Y = [y_1, \ldots, y_m] \in \mathbb{R}^{m \times q} \) represents the \( q \) linear mixtures of \( S \).

Let us denote \( m \) segments or patches of the sources \( s_j \) by \( \Pi_i(s_j) \in \mathbb{R}^r \) with \( r \ll n \) for \( i = 1, \ldots, m \) and \( j = 1, \ldots, p \). All source segments are assumed to have some sparse representations with respect to an unknown but common dictionary \( \Psi \in \mathbb{R}^{r \times s} \), i.e.

\[ \Pi_i(s_j) = \Psi x_{ij}. \]

with sparse coefficients \( x_{ij} \in \mathbb{R}^s \) or, using a more compact notation, as \( \Pi_i(S) = \Psi X_i \) with sparse \( X_i = [x_{i1}, \ldots, x_{ip}] \in \mathbb{R}^{s \times p} \).

Each segment of the mixtures \( \Pi_i(y_{ij}) \in \mathbb{R}^r \) is compressively sensed by a sampling matrix \( \Phi_{ik} \in \mathbb{R}^{r \times s} \) for \( i = 1, \ldots, m \) and \( k = 1, \ldots, q \). Thus, a compressively sensed observation \( \hat{y}_{ik} \in \mathbb{R}^r \) of the \( k \)-th mixture is constructed as

\[ \hat{y}_{ik} = \Phi_{ik} y_{ik} = \Phi_{ik} \Psi X_i a_k. \]

The problem considered in this work can be stated as follows. Given the compressively sensed observations \( \hat{y}_{ik} \in \mathbb{R}^r \), for \( i = 1, \ldots, m \) and \( k = 1, \ldots, q \), and their corresponding sampling matrix \( \Phi_{ik} \in \mathbb{R}^{r \times s} \), estimate the mixing matrix \( A \in \mathbb{R}^{p \times q} \), the sparse representations \( \chi := \{X_i\}_{i=1}^m \in (\mathbb{R}^{s \times q})^m \), and the common dictionary \( \Psi \in \mathbb{R}^{r \times s} \).

By taking the sparse model error and the CS observation error into account, we propose the following cost function

\[ F(S, A, \Psi, \chi) := \sum_{i=1}^m \sum_{j=1}^p \left( \|x_{ij}\|_1 + \lambda_1 \|\Pi_i(s_j) - \Psi x_{ij}\|_2^2 \right) \]

\[ + \sum_{i=1}^m \sum_{k=1}^q \frac{\lambda_2}{2} \|\Phi_{ik} \Pi_i(Sa_k) - \hat{y}_{ik}\|_2^2, \]

where the scalars \( \lambda_1, \lambda_2 \in \mathbb{R}^+ \) weight the sparse model error and the CS observation error, and balance these errors against the sparsity term.

In this work, we employ an alternating scheme for minimizing \( F \), which iterates the following three steps: (i) By fixing \( (A, S) \), the parameters \( (\Psi, \chi) \) are updated using any dictionary learning algorithm, such as K-SVD; (ii) With the parameters \( (S, \Psi, \chi) \) being frozen, the mixing matrix \( A \) is updated by minimizing

\[ f_1(A) := \sum_{i=1}^m \sum_{k=1}^q \|\Phi_{ik} \Pi_i(Sa_k) - \hat{y}_{ik}\|_2^2, \]

and (iii) the sources \( S \) are estimated via a minimizing

\[ f_2(S) := \sum_{i=1}^m \sum_{j=1}^p \lambda_1 \|\Pi_i(s_j) - \Psi x_{ij}\|_2^2 \]

\[ + \sum_{i=1}^m \sum_{k=1}^q \frac{\lambda_2}{2} \|\Phi_{ik} \Pi_i(Sa_k) - \hat{y}_{ik}\|_2^2, \]

with fixed \( (A, \Psi, \chi) \). Note, that both functions \( f_1 \) and \( f_2 \) can explicitly been solved using the pseudo-inverse. Due to the well-known scaling ambiguity in the mixing matrix, we renormalize each row of the solutions of \( \hat{A} \) in each step to have Euclidean norm equal to one. A fixed-point analysis will be provided under the assumption that an optimal dictionary \( \Psi \) is found in step (i). We also demonstrate the performance of our algorithm on synthetic and real data.

REFERENCES


Double Blind Separation of Compressively Sensed Signals

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