Analysis Based Blind Compressive Sensing

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Abstract—In this abstract we address the problem of blindly reconstituting compressively sensed signals by exploiting the co-sparse analysis model. In the analysis model, it is assumed that a signal multiplied by an analysis operator results in a sparse vector. We propose an algorithm that learns the operator adaptively during the reconstruction process. The arising optimization problem is tackled via a geometric conjugate gradient approach.

I. INTRODUCTION

In recent years, Compressive Sensing (CS) has influenced many fields in signal processing. Basically, the theory states that if an unknown signal \( s \in \mathbb{R}^n \) can be sparsely represented, only a few \( m < n \) linear and non-adaptive measurements \( y \in \mathbb{R}^m \) of the signal suffice to accurately reconstruct it. Denoting the measurement vectors by \( \phi_i \in \mathbb{R}^n \) for \( i=1,\ldots,m \), the measurement process can be compactly written as

\[ y = [\phi_1, \ldots, \phi_m]^T s + z = \Phi s + z, \]

where \( \Phi \in \mathbb{R}^{m \times n} \) is the measurement matrix, and \( z \in \mathbb{R}^m \) constitutes possible sampling errors.

In this work, we apply the co-sparse analysis approach to find an estimation \( s^* \) of \( s \) given \( \Phi, y \), and an error model for \( z \). Its underlying assumption is that a signal multiplied by an analysis operator \( \Omega \in \mathbb{R}^{k \times n} \) with \( k \geq n \) results in a sparse vector \( \Omega s \in \mathbb{R}^k \). If \( g : \mathbb{R}^k \rightarrow \mathbb{R} \) denotes a function that measures sparsity, the analysis model assumption is exploited via

\[ s^* = \arg \min_{s \in \mathbb{R}^n} g(\Omega s) \quad \text{s.t.} \quad p(\Phi s - y) \leq \epsilon, \]

with \( \epsilon \in \mathbb{R}^+_{\uparrow} \) being an estimated upper bound of the noise energy and \( p(\cdot) \) reflecting the noise model. Typically, for i.i.d. Gaussian noise, \( p(\cdot) \) is the squared \( \ell_2 \)-norm.

The co-sparse analysis model has particularly proven useful in image reconstruction tasks, e.g. [1], [2] by employing a finite difference operator that approximates the image gradient, known as TV-norm regularization. In [3], [4] it is shown, that the reconstruction quality can be improved by using an analysis operator, which has been learned a priori on the basis of certain image classes. This prompted us to adaptively learn an analysis operator simultaneously to the reconstruction. Regarding the compressive sensing setting, our approach can be considered as an analysis based Blind Compressive Sensing (BCS) [5] problem. Although the present work straightforwardly extends to general signals, we restrict ourselves in the following to compressively sensed images.

II. PROBLEM DESCRIPTION AND RESULTS

Our goal is to simultaneously find an analysis operator \( \Omega^* \in \mathbb{R}^{k \times n} \) with \( k \geq n \) together with the reconstructed signal \( s^* \in \mathbb{R}^N \) that solves a problem related to (2).

We follow [3] and employ a geometric Conjugate Gradient (CG) method on the oblique manifold \( \text{OB}(n,k) \) together with the penalties for regularizing \( \Omega \) which are motivated in [3]. More precisely, \( \Omega^T \)

is restricted to be an element in \( \text{OB}(n,k) \), full rank of \( \Omega \) is enforced by employing the penalty function

\[ h(\Omega) := -\frac{1}{n \log(n)} \log \det \left( \frac{1}{\gamma} \Omega^T \Omega \right), \]

and the mutual coherence of the analysis operator is controlled via the logarithmic barrier function of the atoms’ scalar products, namely

\[ r(\Omega) := -\sum_{1 \leq i < j \leq k} \log(1 - (\omega_i^T \omega_j)^2), \]

where \( \omega_i \) denotes the transposed of the \( i^{th} \)-row of \( \Omega \). Note, that in practice, the analysis operator has to be applied to local image patches rather than to the whole image. We denote the binary \( (n \times N) \) matrix that extracts the patch centered at the \((r,c)\) pixel by \( P_{rc} \). Furthermore, the matrix \( M \in \mathbb{R}^{n \times n} \) subtracts the mean of each image patch. Summarizing, this leads to the optimization problem

\[ (\Omega^*, s^*) = \arg \min_{\Omega \in \mathbb{R}^{k \times n}, \ s \in \mathbb{R}^N} \left( \frac{1}{2M} \sum_{(r,c)} g(\Omega M P_{rc} s)^2 \right) \]

\[ + \eta p(\Phi s - y) + \frac{1}{2} \left( \gamma h(\Omega) + \kappa r(\Omega) \right), \]

with the measurement matrix \( \Phi \in \mathbb{R}^{m \times N} \) and the measurements \( y \in \mathbb{R}^m \). The parameter \( \eta \in \mathbb{R}^+ \) weights the fidelity of the solution to the measurements and the parameters \( \gamma, \kappa \in \mathbb{R}^+ \) control the influence of the two constraints. The scalar \( B \) denotes the number of extractable patches. The advantages of our approach are as follows. (i) The learning process allows to adaptively find an adequate operator, without the necessity of training \( \Omega \) prior to the reconstruction process. (ii) Different noise models are handled by simply exchanging the data fidelity term.

The geometric CG-method for tackling (5) is applied for the problem of reconstructing compressively sensed images. We present experiments, where we outperform state-of-the-art TV-norm regularized reconstruction methods. For example, our achieved PSNR for the reconstruction of the famous Barbara image from \( M = N/4 \) measurements, corrupted by additive white Gaussian noise with standard deviation \( \sigma = 5.1 \) reads 29.79 dB, whereas the reconstruction accuracy of NESTA [6] is 24.71 dB.

REFERENCES